DISTRIBUTION OF LIMIT CYCLES IN QUADRATIC SYSTEMS

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I will present a simplified version of the proof by Zhang Pingguang [1] that in any quadratic system of the type

$$\begin{cases} \dfrac{dx(t)}{dt} = P_2(x(t), y(t)) = a_{00} + a_{10}x(t) + a_{01}y(t) + a_{20}x^2(t) + a_{11}x(t)y(t) + a_{02}y^2(t) \\ \dfrac{dy(t)}{dt} = Q_2(x(t), y(t)) = b_{00} + b_{10}x(t) + b_{01}y(t) + b_{20}x^2(t) + b_{11}x(t)y(t) + b_{02}y^2(t) \end{cases}$$

where $(x, y) \in \mathbb{R}^2$, $t \in \mathbb{R}$, $a_{ij}, b_{ij} \in \mathbb{R}$, only $(n, 0)$ or $(n, 1)$ (where $n \geq 0$) distributions of limit cycles can occur. This result was obtained more than 20 years ago and is one of the fundamental results for quadratic systems and the related 16^{th} Hilbert's problem. The proof is based on a transformation to a Liénard system where the uniqueness of limit cycles is established with the aid of an extension of the Zhang Zhifen theorem. In the proof essentially no complicated calculations are needed and geometrical arguments combined with properties of polynomial functions are sufficient to complete the proof.

[1] Zhang Pingguang, *On the distribution and number of limit cycles for quadratic systems with two foci*, Qual. Theory of Dyn. Sys., 3, 47(2002), 437–463.