TRANSITION BETWEEN MONOSTABILITY AND BISTABILITY OF A GENETIC TOGGLE SWITCH IN ESCHERICHIA COLI

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In this paper, we investigate a genetic toggle switch in Escherichia Coli, which models an artificial double-negative feedback loop with two mutually repressors. This model is a planar differential system with three parameters, one of which is an integer power $n \geq 1$, in the case that repressors 1 and 2 multimerize with n and 1 subunits respectively and its equilibria are decided by a polynomial of degree $n + 1$. Since one hardly solves such a polynomial equation, a known result on bistability was given by omitting some small terms under the assumption that the promoters are strong and the expression ration between the ON state and the OFF state is large. In this paper, determining distribution of zeros qualitatively for the polynomial of high degree, we analytically discuss on the system without the assumption and completely give qualitative properties for all equilibria, which corrects the known results of bistability. Furthermore, we prove that there may occur in the system a codimension 2 bifurcation, called cusp bifurcation, which is a collision of two saddle-node bifurcations and manifests the transition between bistability and monostability. We exhibit the global dynamics of repressors in various cases by analyzing equilibria at infinity and providing nonexistence of closed orbits.

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