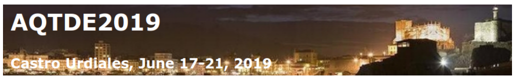

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On the number of inseparable leaves of chordal polynomial systems in the plane

Francisco Braun

Universidade Federal de São Carlos (São Carlos, Brazil)

A system of differential equations in the plane $\dot{x} = P(x, y)$, $\dot{y} = Q(x, y)$ is called *chordal* when P and Q have no common (real) zeros. It is well known that the topological classification of chordal systems depends only on the inseparable leaves (orbits) of the system and their distribution on the plane. Recall that two orbits γ_1 and γ_2 of a chordal system are called *inseparable* if for each transversal sections S_1 and S_2 of γ_1 and γ_2, respectively, there exists another orbit intersecting S_1 and S_2. For polynomial chordal systems of degree n, i.e. P and Q are polynomials and n is the maximum between the degrees of P and Q, there are finitely many inseparable leaves. We denote by $s(n)$ the maximal number of inseparable leaves that a polynomial chordal system of degree n can have. In this lecture we briefly survey the known results on lower and upper bounds of $s(n)$ and present a class of polynomial chordal *Hamiltonian* systems that improves the lower bound of $s(n)$ for all $n \geq 4$.

This is a joint work with F. Fernandes.

Some aspects of integrability in 2 and 3 dimensions

Colin Christopher

University of Plymouth (Plymouth, UK)

We survey some aspects of the integrability of two and three-dimensional polynomial vector fields with some indications of further directions for research.

Asymptotic lower bounds on Hilbert numbers using canard cycles

Peter De Maesschalck

Hasselt University (Hasselt, Belgium)

We present a construction using singular perturbations of planar polynomial vector fields of large degree N where the number of limit cycles grows asymptotically like $CN^2 \log N$, for some constant C. We compare with results in the literature.

This is a joint work with M. J. Álvarez, B. Coll, and R. Prohens.

Non-integrability criteria for polynomial differential systems in $\mathbb{C}^2$

Jaume Giné

Universitat de Lleida (Lleida, Spain)

First we revisit the integrability problem for polynomial differential systems in $\mathbb{C}^2$. Next we give two recently new criteria for determining Puiseux Weierstrass non-integrability of some polynomial differential systems in $\mathbb{C}^2$. These criteria use solutions of the form $y = f(x)$ of the differential system in the plane and their associated cofactors, where $f(x)$ is a formal power series.

Cyclicity of canard cycles with hyperbolic saddles located away from the critical curve

Renato Huzak

Hasselt University (Hasselt, Belgium)

The goal of our paper is to study limit cycles in smooth planar slow-fast systems, Hausdorff close to canard cycles containing hyperbolic saddles located away from the critical (or slow) curve. Our focus is on a broad class of smooth slow-fast systems with a Hopf breaking mechanism (often called a generic turning point). Such canard cycles naturally occur in predator-prey systems with Holling type II and IV response functions and with a small predator's death rate. The study of canard cycles is also relevant for the Hilbert's 16th problem. We primarily focus on the canard cycles with one hyperbolic saddle (located away from the slow curve) and we allow isolated singularities in the slow dynamics.

On the integrability of the N–dimensional differential systems

JAUME LLIBRE
Universitat Autònoma de Barcelona (Bellaterra, Spain)

First we present some new results on the complete integrability of the N–dimensional differential systems, based on the Nambu bracket and the Jacobi multiplier. Second we provide two improvements to the classical Jacobi Theorem on the complete integrability of the N–dimensional differential systems having $N-2$ independent first integrals and a Jacobi multiplier. Third we present new results on the complete integrability of the 3–dimensional differential Lotka–Volterra differential systems and on their Jacobi multipliers. We wil use the techniques of the book [1].

This is a joint work with R. Ramírez and V. Ramírez.

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On the number of polynomial solutions of Bernoulli and Abel polynomial differential equations

FRANCESC MAÑOSAS
Universitat Autònoma de Barcelona (Bellaterra, Spain)

We determine the maximum number of polynomial solutions of Bernoulli differential equations and of some integrable polynomial Abel differential equations.

We show that the addressed problems can be reduced to know the number of polynomial solutions of a related polynomial equation of arbitrary degree. Then we approach to these equations either applying several tools developed to study extended Fermat problems for polynomial equations, or reducing the question to the computation of the genus of some associated planar algebraic curves.

This is a joint work with A. Cima and A. Gasull.

The tennis racket effect and the impossible skateboard flip

Pavao Mardesic

Université de Bourgogne (Dijon, France)

We study the phenomenon observed when throwing a tennis racket in the air so that its handle performs a full turn. What one observes is a flip of the tennis racket head. The phenomenon is described by examining Euler's equations, which lead to elliptic integrals. We establish how the phenomenon depends on different parameters.

This is a joint work with D. Sugny and L. Van Damme.

Complex Cellular Structures

Dmitry Novikov

Weizmann Institute of Science (Rehovot, Israel)

Real semialgebraic sets admit so-called cellular decomposition, i.e. representation as a union of cells homeomorphic to cubes. Attempts to build a straightforward complex holomorphic generalization of this construction meet difficulties related to inner metric properties of holomorphic sets, absent in real case. I will explain these difficulties and propose a complexification of the real cell decomposition for analytic and semialgebraic sets, motivated by classical Resolution of Singularities and Yomdin–Gromov C^r-parameterization results.

This is a joint work with G. Binyamini.

Classifications of Dulac germs

Maja Resman

University of Zagreb, (Zagreb, Croatia)

We consider one-dimensional germs which admit asymptotic expansion in power-logarithmic scale (the Dulac maps). We discuss formal and analytic classification of such maps, related to the question of formal and analytic embedding as a time-one map in a vector field. As a crucial step, we construct a (sectorial and formal) Fatou coordinate of a Dulac germ. The research of such germs is motivated by the question of cyclicity for hyperbolic polycycles in planar vector fields.

This is a joint work with P. Mardešić, J.P. Rolin, and V. Županović.

Qualitative studies of some biochemical models

Valery Romanovsky

CAMTP and University of Maribor (Maribor, Slovenia)

We discuss how the methods of computational algebra can be used for the investigation of higher dimensional systems related to certain reaction kinetics networks. We first present an approach to find invariant surfaces and first integrals, which can be also used to determine center manifolds of the system. Then, an approach to investigate Hopf and Bautin bifurcations on a center manifold is presented.

Optimal network architectures for metapopulations with heterogeneous diffusion

Pedro Torres

Universidad de Granada (Granada, Spain)

In this talk we will introduce the notion of metapopulation as a population spatially distributed in a fragmented habitat composed by interconnected patches. A simple mathematical model is a system of linearly coupled logistic equations.

Then we analise the problem of finding the best network architecture among the patches that maximizes the total population. It turns out that the answer to this problem depends on the mobility degree of the individuals, which is taken as a free parameters. Some extensions of the essential ideas to models in Epidemiology are presented as well.

Reduction of chemical reaction networks

Sebastian Walcher

RWTH Aachen (Aachen, Germany)

Important classes of chemical reaction networks (CRN) can be described and analyzed by parameter dependent polynomial ordinary differential equations. Such systems may a priori be high dimensional, hence there is theoretical and practical interest in reducing dimension. Some heuristic approaches from biochemistry (such as the quasi-steady state assumptions for certain chemical species), and the observation that different reactions in a given network may run in very different time scales, naturally suggest to work in the framework of singular perturbation theory. In the talk we will discuss computational and practical problems related to this approach:

1. For general reaction networks there is no “small parameter” given a priori, and the first task is to identify those critical parameter values, which provide singular perturbation scenarios upon small perturbations.

2. Given a system with “small parameter”, one will not in general have a separation of variables into fast and slow ones; thus a coordinate-independent reduction procedure is needed.

We give solutions to both problems (for general polynomial systems), using methods from algorithmic algebra, and present some applications. We will also discuss the extension to multiple time scales and the special properties of CRN which help make the method more feasible.

The lowest upper bound on the number of zeros of Abelian integrals

Dongmei Xiao

Shanghai Jiao Tong University (Shanghai, China)

In this talk, we will introduce a new method to estimate the lowest upper bound of the number of isolated zeros of Abelian integrals. Some algebraic criteria are obtained for the number of isolated zeros of Abelian integrals along energy level ovals of potential systems. As applications of our main results, we study three kinds of Abelian integrals along algebraic or non-algebraic level ovals, obtain the algebraic criteria on the Abelian integrals having Chebyshev property with accuracy one, simplify some known proof on the cyclicity of quadratic reversible centers, and give all the configurations of limit cycles from Poincaré bifurcation of two quadratic reversible systems with two centers, one of which has a non-algebraic first integral with logarithmic function.

This talk is based on the joint works with Changjian Liu.

A soft quasi-invariant of Fuchsian equations on the complex projective line

Sergei Yakovenko

The Weizmann Insttitute of Science (Rehovot, Israel)

A linear higher order homogeneous differential equation with rational coefficients on the projective line (Riemann sphere) necessarily has singular points, whose location and characteristic numbers largely control oscillation (the number of isolated zeros, properly counted) of solutions of the equation. There is yet another, less visible characteristic that affects the oscillatory behavior: proximity to a singularly degenerated equation. Such degeneracy occurs when the coefficient before the leading term (highest order derivative) becomes very small relative to the non-leading terms. One could consider, as the simplest example, the equation $y'' - My = 0$, where M is a positive parameter that grows to $+\infty$: the number of its isolated zeros on any finite real interval (away from singular point) grows to infinity. In the talk we will introduce this characteristic (called slope) and discuss its behavior, in particular, its explicit boundedness for families of Fuchsian equations.

Studying the dynamics of some Lagrangian systems by nonlocal constants of motion

Gaetano Zampieri

Università di Verona (Verona, Italy)

We show a simple general theorem which is a tool that generates nonlocal constants of motion for Lagrangian systems. We review some cases where the constants that we find are useful in the study of the system: the homogeneous potentials of degree -2, the mechanical systems with viscous fluid resistance and the conservative and dissipative Maxwell-Bloch equations of laser dynamics. We also add a new result on explosion in the past for mechanical system with quadratic fluid resistance and bounded potential.

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Dynamics of slow fast predator–prey model of generalized Holling type III

Xiang Zhang

Shanghai Jiao Tong University (Shanghai, China)

The predator–prey model of generalized Holling type III has existed for a long time, but its complete dynamics is still unknown. In this talk we introduce our results on this model via geometric singular perturbation, where we obtain the existence of global stability, canard cycles, relaxation oscillations, canard explosion, homoclinc and hetoroclinic orbits, and the cyclicity of singular slow fast cycles.

Stratification of the dynamics of analytic real vector fields in dimension three

Clementa Alonso-González

Universidad de Alicante (Alicante, Spain)

There is a classical well known result that states that if ξ is a real analytic vector field with an isolated singularity at $0 \in \mathbb{R}^2$ which is not of the *center-focus* type then we can decompose a small enough open neighborhood U of 0 into finitely many *hyperbolic, elliptic or parabolic sectors.* In this talk we present an analogous result for analytic three dimensional real vector fields under non degeneracy conditions.

This is a joint work with F. Sanz.

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Latest advances towards a complete classification of quadratic phase portraits

Joan Carles Artés

Universitat Autònoma de Barcelona (Bellaterra, Spain)

Along the past years, our group has designed a systematic algorithm which, in a reasonable lapse of time, will allow to obtain a complete classification of phase portraits of quadratic differential systems modulo limit cycles. The kernel of the classification is the partition of parameter space in 208 parts where the set of singularities is topologically different from each other. But in order to be sure that this collection of 208 was complete, previously a geometrical classification of singularities in 1765 subsets has been needed, all of them with the help of geometrical invariants.

This is a joint work with J. Llibre, R. Oliveira, A. Rezende, D. Schlomiuk, and N. Vulpe.

Center problem for a class degenerate systems

María Diaz
Universidad de Huelva (Huelva, Spain)

The center-focus problem or the monodromy problem consists in characterizing when an equilibrium point of a polynomial system is a focus or a center. Once we know the equilibrium is monodromic, it appears another classic, called *the center problem* or *the stability problem*: How could we differentiate a center from a focus?. If the linear part is non degenerate, the Poincaré-Lyapunov's method solves the center problem (see [6,7,8,9]), in this case the origin of the perturbation of a lineal center is a center if, and only if, it is analytically integrable. Another known result, (see [2]), is the characterization of the centers of a nilpotent systems, if the origin of a nilpotent system is monodromic, then it will be a center if, and only if, the system is orbitally reversible. By other hand, Ilyashencko [5] and Ecalle [4] proved each one by his own that the origin of polynomial systems can not be an accumulation point of limit cycles. In this way if the origin from a polynomial system is monodromic, or it is a center or a focus. Another method to solve the center problem is the Bautin's method, (see [1] for non-degenerate vector fields), it consists in calculating the Poincaré application using a recursive system of linear differential equations. We will use this method to characterizer centers of degenerate systems. Using this method we are able to find the first terms of the Poincaré application. For calculating the focal values we generalize an idea from Briskin, Francoise & Yomdin [3], where we calculate an inverse integrating factor of Abel's equation generalized corresponding to the system.

This is a joint work with A. Algaba, C. García, and J. Giné.

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A two-parameter family of vector fields with inclination-flip and orbit-flip homoclinic connections

María de la Cinta Domínguez-Moreno
Universidad de Huelva (Huelva, Spain)

In this work, we consider the following quadratic two-parameter family of vector fields,

$$\left\{\begin{array}{rcl} \dot{x} &=& a + yz, \\ \dot{y} &=& -y + x^2, \\ \dot{z} &=& b - 4x, \end{array}\right.$$

with $b \neq 0$. The system considered in [4] embeds the above family. We study the Hopf bifurcation of its only equilibrium $E = \left(\frac{b}{4}, \frac{b^2}{16}, \frac{-16a}{b^2}\right)$. From the information obtained in this analysis and by means of methods of numerical continuation

[1], saddle-node of periodic orbits and period-doubling bifurcations as well as homoclinic connections appear. A careful study of the homoclinic orbits determines that the organizing center in a region of the parameter plane where the only equilibrium has three real eigenvalues, is a homoclinic flip bifurcation of the inward twist case **Cin** [2,3]. As far as we know, is the first example of a 3D vector field exhibiting this case, where a chaotic behavior with Smale horseshoes and strange attractors is warranted.

This is a joint work with A. Algaba, M. Merino, and A. J. Rodríguez-Luis.

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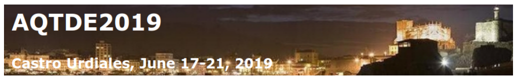


The center cyclicity of the Lorenz, Chen and Lü systems

Isaac Garcia

Universitat de Lleida (Lleida, Spain)

his work provides upper bounds on the cyclicity of the centers on center manifolds in the well-known Lorenz family, and also in the Chen and Lü families. We prove that at most one limit cycle can be made to bifurcate from any center of any element of these families, perturbing within the respective family, with the exception of one specific Lorenz system where the cyclicity increases. We also show that this bound is sharp.

This is a joint work with S. Maza and D. S. Shafer.

Dulac map and time in families of hyperbolic saddles

David Marín

Universitat Auiònoma de Barcelona (Barcelona, Spain)

We describe uniform asymptotic expansions for the Dulac map and time associated to families of hyperbolic saddles between arbitrary transverse sections. We provide explicit formulae for the first coefficients in the Roussarie's monomial scale by introducing a new integral operator (generalizing Mellin transform) which renormalizes divergent improper integrals. We illustrate this method by computing the three first coefficients of the asymptotic expansion of the period function associated to the quadratic centers in the Loud family.

This is a joint work with J. Villadelprat.

Resonance of isochronous oscillators

David Rojas
Universitat de Girona (Girona, Spain)

An oscillator is called isochronous if all motions have a common period. When the system is forced by a time-dependent perturbation with the same period the dynamics may change and the phenomenon of resonance can appear. In this context, resonance means that all solutions are unbounded. The theory of resonance is well known for the harmonic oscillator and we extend it to nonlinear isochronous oscillators.

This is a joint work with R. Ortega.

Real analytic vector fields with first integral and separatrices

Fernando Sanz
Universidad de Valladolid (Valladolid, Spain)

We prove that a germ of analytic vector field at $(\mathbb{R}^3, 0)$ with a non-constant analytic first integral always has a real formal invariant curve. We provide an example which shows that such a vector field does not necessarily have a real analytic invariant curve.

This is a joint work with R. Mol.

Limit cycle bifurcations near an elementary center and a homoclinic loop

Yun Tian
Shanghai Normal University (Shanghai China)

In this talk, we mainly focus on bifurcations of limit cycles around the boundaries (an elementary center or a homoclinic loop) of an period annulus in planar integrable systems under small perturbations. We present a method of high-order analysis for Hopf bifurcation, and give a new method to compute the coefficients of the expansion of Menikov functions around $h = h_s$, where a homoclinic loop is defined by $H = h_s$.

Perturbations of several identical harmonic oscillators, all of whose orbits are periodic: truly non-linear vs. rigid dynamics

Massimo Villarini
Università degli Studi di Modena e Reggio Emilia (Modena, Italy)

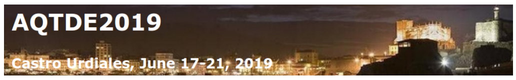
The starting point of the talk is the classical Poincaré-Lyapunov Centre Theorem in planar dynamical systems: a non-degenerate analytic centre is always linearizable. In the case of a family of non-degenerate centres perturbing the harmonic oscillator, the Poincaré-Lyapunov Centre Theorem is a rigidity results-the dynamics of the perturbed systems is orbitally equivalent to the linear one. In the case of n, n greater or equal two, identical harmonic oscillators the analogous situation is that of a non-degenerate multicentre: the linear part of the vector field having a multicentre singular point is the infinitesimal generator of a non-degenerate rotation in the real 2n-dimensional space, and all orbits in a neighbourhood of the singular point are closed. In this case the linearizability result analogous to Poincaré-Lyapunov theorem is valid if the system is real analytic and the period function is locally bounded near the singular point (Urabe-Sibuya around 1955, and M. Brunella-M.V. 1999), but in general this rigidity, or linearizability result is false and the period function can be locally unbounded near the singular point.: An example when $n = 4$ is given in M.V. Erg. Theory Dyn. Syst. 2019. On the other hand rigidity results based on the principle that on has rigidity if a codimension 2 subsystem is kept fixed has been recently obtained (M.V. 2018 arXiv). These results has more general statements and several geometric consequences, but the talk will privilege the dynamicl aproach chosen in this abstract.

Quadratic differential systems possessing invariant ellipses: a complete classification in the space $\mathbb{R}^{12}$

Nicolae Vulpe
Academy of Sciences of Moldova (Chisinau, Republic of Moldova)

Consider the class **QS** of all non-degenerate quadratic quadratic polynomial differential systems. Note that each system in this class can be identified with a point of $\mathbb{R}^{12}$ through its coefficients. We provide necessary and sufficient conditions for a quadratic system in **QS** to have at least one invariant ellipse in terms of its coefficients. Moreover we present affine invariant criteria for a system in **QS** to have an algebraic limit cycle, which is an ellipse. The global bifurcation diagram of the whole class of non-degenerate planar quadratic differential systems possessing at least one invariant ellipse is given. The bifurcation diagram is expressed in terms of invariant polynomial and it is done in the 12-dimensional space of parameters. Therefore, the obtained results can be applied for any family of quadratic systems in this class, given in any normal form.

This is a joint work with R. D. S. Oliveira, A. C. Rezende, and D. Schlomiuk.



Bifurcation of one codimension and structural stability in piecewise smooth vector fields via regularization

Yagor Romano Carvalho

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We consider discontinuous vector fields in the plane $Z = (X, Y)$, given by $Z(x, y) = X(x, y)$ if $xy \geqslant 0$ and $Z(x, y) = Y(x, y)$ if $xy \leqslant 0$, where $X, Y : V \subset \mathbb{R}^2 \to \mathbb{R}^2$ are smooth vector fields and V is a neighborhood of the origin. The set of all these vector fields is denoted by χ^D. We endow χ^D with the product topology. We denote by χ^S the set of all smooth vector fields $X : V \subset \mathbb{R}^2 \to \mathbb{R}^2$. We say that two of these discontinuous vector fields are locally Σ-equivalent if there are neighborhoods around the origin and a homeomorphism that sends trajectories of one vector field to the trajectories of the other, preserving the orientation, and it sends $\Sigma = \{(x, y) \in \mathbb{R}^2 \ ; \ xy = 0\}$ to Σ. A vector field is locally Σ-structurally stable at the origin if there is a neighborhood of this vector field in χ^D such that all vector fields in this neighborhood are locally Σ-equivalent. When $Z \in \chi^D$ is not locally Σ-structurally stable, we say that Z belongs to the bifurcation set. Consider Z in the bifurcation set, there is a codimension one singularity in the origin if it's relatively Σ-structurally stable in the induced topology of the bifurcation set, that is, if given the induced topology of χ^D in the bifurcation set then there is an open set in the bifurcation set such that every field in that open is locally Σ-equivalent to Z and any unfolding of Z and of the fields in this open are weak equivalents, where two unfoldings are weak equivalent if there is a homeomorphic change of parameters, such that, for each correspondence of parameters, their respective fields are locally Σ-equivalent. In this work we consider a particular regularization that is a correspondence that for each $Z \in \chi^D$ associates a field $Z^R \in \chi^S$. Our main results in this work are:

Theorem. If Z is structurally stable at χ^D, then Z^R is structurally stable in χ^S.

Theorem. If Z has a codimension one singularity at the origin, then Z^R has not.

This is a joint work with C. A. Buzzi.

Invariant manifolds of parabolic points with nilpotent part

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Let F be a planar analytic map having a parabolic fixed point with nilpotent part. Given the normal form of F, we provide an algorithm to compute an approximation up to any order of a stable curve associated with this point. Then, we prove the existence of such a curve as an a posteriori result using the parameterization method for invariant manifolds. Concretely, we show that the approximation obtained from the algorithm converges to a parameterization of the invariant curve and we provide the analyticity of the curve in an open set that does not contain the fixed point.

This is a joint work with E. Fontich.

Dicritical foliations and the Zariski's invariant of plane branches

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The problem of the analytic classification of the germs of plane branches was posed by O. Zariski in 1973. His aim was to get a complete description of the moduli space for a fixed equisingularity class of a plane branch in a similiar way as it was done for algebraic curves of a given genus. In this case the objects are the germs of plane branches they play the same role of the algebraic curves and the analytic equivalence subsitute the birational equivalence used in the global problem. The problem for a plane branch stayed open for almost 40 years until A. Hefez and M. E. Hernandes, in 2011, gave the complete solution in [1]. We firstly give a geometric way to compute one of the analytic invariant for plane blanches. In [3] O. Zariski shows that in the case of the plane branch $\mathcal{C}$ is not quasihomogeneous, that is, the plane branch is not analytically equivalent to a branch given by (t^n, t^m) then there exists a term $a_\lambda t^\lambda$ with $a_\lambda \neq 0$ in the Puiseux parametrization of $\mathcal{C}$ where the exponent is the minimum integer that satisfies

$\lambda \notin S$ and $\lambda + n - m \notin S$. This number λ is better known as the *Zariski invariant.* We only consider the case of singular irreducible plane curves with an unique Puisex pair (n, m) with $1 < n < m$ and the g.c.d$(n, m) = 1$. We show that the *Zariski invariant* of a plane branch with a unique Puiseux pair $\mathcal{C}$ is the tangency order between the plane branch and a singular holomporhic foliation in $(\mathbb{C}^2, 0)$ analytically equivalent to the foliation generated locally by $\omega_0 = nxdy - mydx$. In the second part, following the work of R. Peraire [2] we give some examples of singular holomorphic foliations in $(\mathbb{C}^2, 0)$ whose resolution of singularities is given by following the Euclidean algorithm of (n, m) and the last component of the exceptional divisor is dicritical. The analytic curves are the strict transform of irreducible plane curves in the equisingularity class (n, m) with the same *Zariski's invariant.* We study the foliations with this phenomenon.

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Thirty-three small limit cycles in a quintic vector field

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One of problems that remains open is the second part of the 16th Hilbert's problem that consist in to determine the maximal number (named $H(n)$) of limit cycles, and their relative positions, of a planar polynomial systems of degree n. We are interested here in the local version of this Hilbert's 16th problem, that is, to provide the number $M(n)$ of small amplitude limit cycles bifurcating from a point of center-focus type. Bautin (1954) proved that $M(2) = 3$. Sibirskii (1965) proved that for cubic systems without quadratic terms have no more than five limit cycles bifurcating from one critical point. Zoladek (1995, 2016) found an example where eleven limit cycles could be bifurcated from a single critical point of a cubic system and Christopher (2006), studying only the Taylor developments of the Liapunov quantities, provides a simpler proof of the existence of a cubic system with 11 limit cycles. We prove that $M(5)$ is bigger than or equal to 33. More concretely, we present a quintic Darboux center such that 33 small limit cycles bifurcate from the origin via a degenerated Hopf bifurcation, when we perturb in the class of polynomial vector fields of degree five. We remark that this lower bound coincide with the value, $M(n) = n^2+3n-7$, conjectured to be the maximum value by Giné (2012). The computations have been done using a generalization of the parallelization procedure, introduced by Liang and Torregrosa (2015), for finding the higher order terms in the perturbation parameters of the coefficients of the return map.

This is a joint work with J. Torregrosa.

An algorithm for providing the normal forms of three-dimensional quasi-homogeneous polynomial differential systems

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Quasi-homogeneous systems, and in particular those 3-dimensional, are currently a thriving line of research. But a method for obtaining all fields of this class is not yet available. The weight vectors of a quasi-homogeneous system are grouped into families. We found that the maximal three-dimensional quasi-homogeneous systems have the property of having only one family with minimum weight vector. This minimum vector is unique to the system, thus acting as identification code. We develop an algorithm that provides all normal forms of maximal three-dimensional quasi-homogeneous systems for a given degree. All other three-dimensional quasi-homogeneous systems can be trivially deduced from these maximal systems. We also list all the systems of this type of degree 2 using the algorithm. With this algorithm we make available to the researchers all three-dimensional quasi-homogeneous systems.

This is a joint work with B. García, J. Llibre, J. S. Pérez del Río.

Toric type foliations in dimension three and invariant surfaces

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Few years ago, René Thom asked the following question: *Is there always an invariant surface for a given codimension one foliation on* $(\mathbb{C}^n, 0)$*?* C. Camacho and P. Sad proved, in 1982, that any foliation of $(\mathbb{C}^2, 0)$ has an invariant curve. For greater dimension, there are results of Cano-Cerveau and Cano-Mattei, that give a positive answer to Thom's question for non-dicritical foliations. However, when the existence of dicritical components of the exceptional divisor is allowed, Jouanolou gives a collection of conic foliations on $(\mathbb{C}^3, 0)$ without invariant surface. What we present in this poster, is a result of existence of invariant surface for, dicritical or not, toric type foliations on $(\mathbb{C}^3, 0)$. By toric type we mean the existence of a combinatorial process of reduction of singularities. This is a slightly stronger condition than being a Newton non-degenerate foliation (in the classical sense in terms of Newton polyhedra). In order to prove this three-dimensional result of local nature, we state and prove the following global result in dimension two: "The isolated invariant curves of toric type foliations on compact toric surfaces are closed curves".

Towards combinatorial monomialization of generalized power series

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The basic definitions and properties of *Generalized Power Series* were introduced by L. van Den Dries and P. Speissegger in [2]. They are power series in any number of variables with positive real exponents whose support is contained in the product of well–ordered subsets of $\mathbb{R}_{\geq 0}$. It has been shown that these power series satisfies the properties of *o*-minimality, *cf.* [2]. Moreover, they define functions whose natural domains are orthants in $\mathbb{R}^n$, analytic in the interior and continuous at the boundary. Thus, the category of real generalized analytic manifolds is well defined. Besides, these objects arise naturally in many different settings. For example, as integral curves of linear differential equations: One can observe that the phase curves associated to the solutions of the differential equation $\dot{x} = Ax$, where A is the diagonal matrix $A = \mathrm{Diag}(1, \lambda)$, $\lambda \in \mathbb{R} \setminus \mathbb{Q}$ is a positive non rational number, are parametrized by the family of curves $\{y = c\,x^{\lambda}\}$, $c \in \mathbb{R}$. Many reasonable questions linked with the resolution of singularities in the analytic and algebraic geometry are posed to this new category; *e.g.* local or global monomialization for generalized analytic functions and morphisms. Local monomialization was obtained by R. Martín Villaverde, J. P. Rolin and F. Sanz Sánchez in [3] in general following the classical ideas for the analytic (*standard*) case *v.* [1] and giving a good definition of the blow–up morphisms in the category of real generalized analytic manifolds. In order to obtain a global monomialization result; *i.e.* using global centres of blowing-up, one needs to overcome the difficulty that appears in this category: there are different non-isomorphic ways of blowing-up and, moreover, there can be regular sub-varieties which cannot be centres of permissible blow-ups. Our goal is to present a global monomialization result for real generalized analytic functions in three real variables and the very first step is to get a monomial presentation respect to the generalized variables, that is what we have called *combinatorial monomialization*.

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Integrability and linearizability of a family of three-dimensional quadratic systems

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We present necessary conditions for the local integrability and linearizability at the origin of three dimensional vector fields with $(1 : -2 : 1)$ resonance. We choose for our investigation a nine–parameter family with quadratic nonlinearities. We finally prove the sufficiency of these conditions using Darboux's method, the existence of Jacobi multiplier and the linearizability of a node in two variables. Additionally, in most of the cases we exhibit two independent Darboux first integrals. For a particular subfamily with three parameters we prove that there are no polynomial first integral.

This is a joint work with W. Aziz and A. Amen.

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Correspondence between Godbillon-Vey sequence and Françoise algorithm

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In the work [1] we study local integrable foliations and their deformations. We introduce the so-called Françoise algorithm for computing the first nonzero Melnikov function for the displacement map, and describe what we call *the Godbillon-Vey sequence for a deformation.* We establish the correspondence between these objects and give some results on the integrability of the deformation in terms of the length of the sequences.

This is a joint work with P. Mardesic, D. Novikov, and L. Ortiz-Bobadilla.

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The number of limit cycles from a cubic center by the Melnikov function of any order

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In this paper, we consider the system $\dot{x} = y(1+x)^2 - \varepsilon P(x, y)$, $\dot{y} = -x(1+x)^2 + \varepsilon Q(x, y)$ where $P(x, y)$ and $Q(x, y)$ are arbitrary quadratic polynomials. We study the upper bound of the number of limit cycles bifurcating from the periodic orbits by using the Melnikov function of any order. We prove that the upper bound of the number of limit cycles is 3 and reached.

This is a joint work with P. Yang.

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