GLOBAL PHASE PORTRAITS OF THE QUARTIC UNIFORM ISOCHRONOUS CENTERS

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Abstract
We classify the global phase portraits in the Poincaré disc of all quartic polynomial differential systems with a uniform isochronous center.

Definitions
Definition 1. Let $p \in \mathbb{R}^2$ be a center of a differential polynomial system in $\mathbb{R}^2$. We say that $p$ is an isochronous center if it is a center having a neighborhood such that all the periodic orbits in this neighborhood have the same period.

Definition 2. We say that $p$ is a uniform isochronous center if the system, in polar coordinates $x = r \cos \theta$, $y = r \sin \theta$, takes the form $r = G(\theta, r)$, $\theta = k$, $k \in \mathbb{R} \setminus \{0\}$.

Phase portraits of the uniform isochronous centers of degree $< 4$

Degree 1
\[
\begin{align*}
\dot{x} &= -y, \\
\dot{y} &= x
\end{align*}
\]

Degree 2 [4]
\[
\begin{align*}
\dot{x} &= -y + x^2, \\
\dot{y} &= x + xy
\end{align*}
\]

Degree 3 [1]
\[
\begin{align*}
\dot{x} &= -y + x^2y, \\
\dot{y} &= x + xy^2
\end{align*}
\]
\[
\begin{align*}
\dot{x} &= -y + x^3 + Axy^2, \\
\dot{y} &= x + xy + Axy^2
\end{align*}
\]

Main results
Consider the family of differential systems with a uniform isochronous center at the origin
\[
\begin{align*}
\dot{x} &= -y + xf(x, y), \\
\dot{y} &= x + yf(x, y)
\end{align*}
\]
where $f(x, y)$ is a polynomial of degree 3 with $f(0, 0) = 0$. We classify the global phase portraits in the Poincaré disc of the vector fields associated to system (1).

Theorem 1. [2] Let (1) be a differential system with a uniform isochronous center at the origin and such that its nonlinear part is not homogeneous. Then the global phase portrait of (1) is topologically equivalent to one of the following 11 phase portraits:

Theorem 2. [3] Let $f(x, y)$ be a cubic homogeneous polynomial in system (1). Then (1) always has a uniform isochronous center at the origin and its global phase portrait is topologically equivalent to one of the following 3 phase portraits:

Discussion and future works
The classification of the global phase portraits in the Poincaré disc of the quartic uniform isochronous centers consists of 13 topologically different phase portraits. In [1] we provided the global phase portraits of the cubic uniform isochronous centers.

Our next step will be to provide a similar classification for the quintic uniform isochronous centers and also investigate the bifurcation of limit cycles from the periodic solutions of these centers.

References

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