# GLOBAL PHASE PORTRAITS OF THE QUARTIC UNIFORM ISOCHRONOUS CENTERS

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## Abstract

We classify the global phase portraits in the Poincaré disc of all quartic polynomial differential systems with a uniform isochronous center.

## Definitions

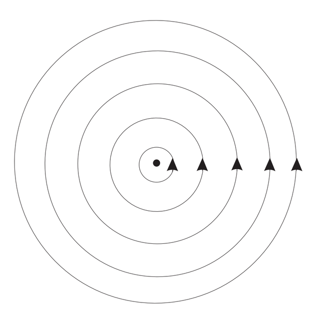
**Definition 1.** Let $p \in \mathbb{R}^2$ be a center of a differential polynomial system in $\mathbb{R}^2$. We say that $p$ is an *isochronous center* if it is a center having a neighborhood such that all the periodic orbits in this neighborhood have the same period.

**Definition 2.** We say that $p$ is a *uniform isochronous center* if the system, in polar coordinates $x = r\cos\theta$, $y = r\sin\theta$, takes the form $\dot{r} = G(\theta, r)$, $\dot{\theta} = k$, $k \in \mathbb{R} \setminus \{0\}$.

## Phase portraits of the uniform isochronous centers of degree $< 4$

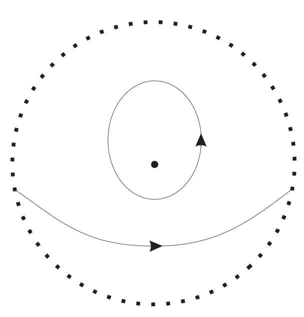
### Degree 1

$$\begin{cases} \dot{x} = -y, \\ \dot{y} = x \end{cases}$$



### Degree 2 [4]

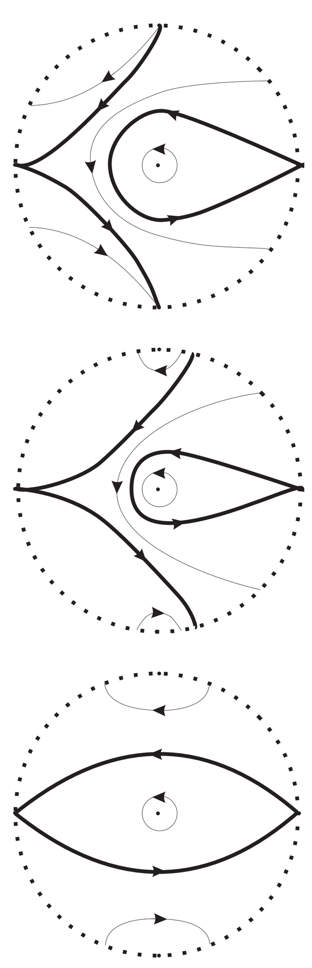
$$\begin{cases} \dot{x} = -y + x^2, \\ \dot{y} = x + xy \end{cases}$$



### Degree 3 [1]

$$\begin{cases} \dot{x} = -y + x^2y, \\ \dot{y} = x + xy^2 \end{cases}$$

$$\begin{cases} \dot{x} = -y + x^2 + Ax^2y, \\ \dot{y} = x + xy + Axy^2 \end{cases}$$



## Main results

Consider the family of differential systems with a uniform isochronous center at the origin

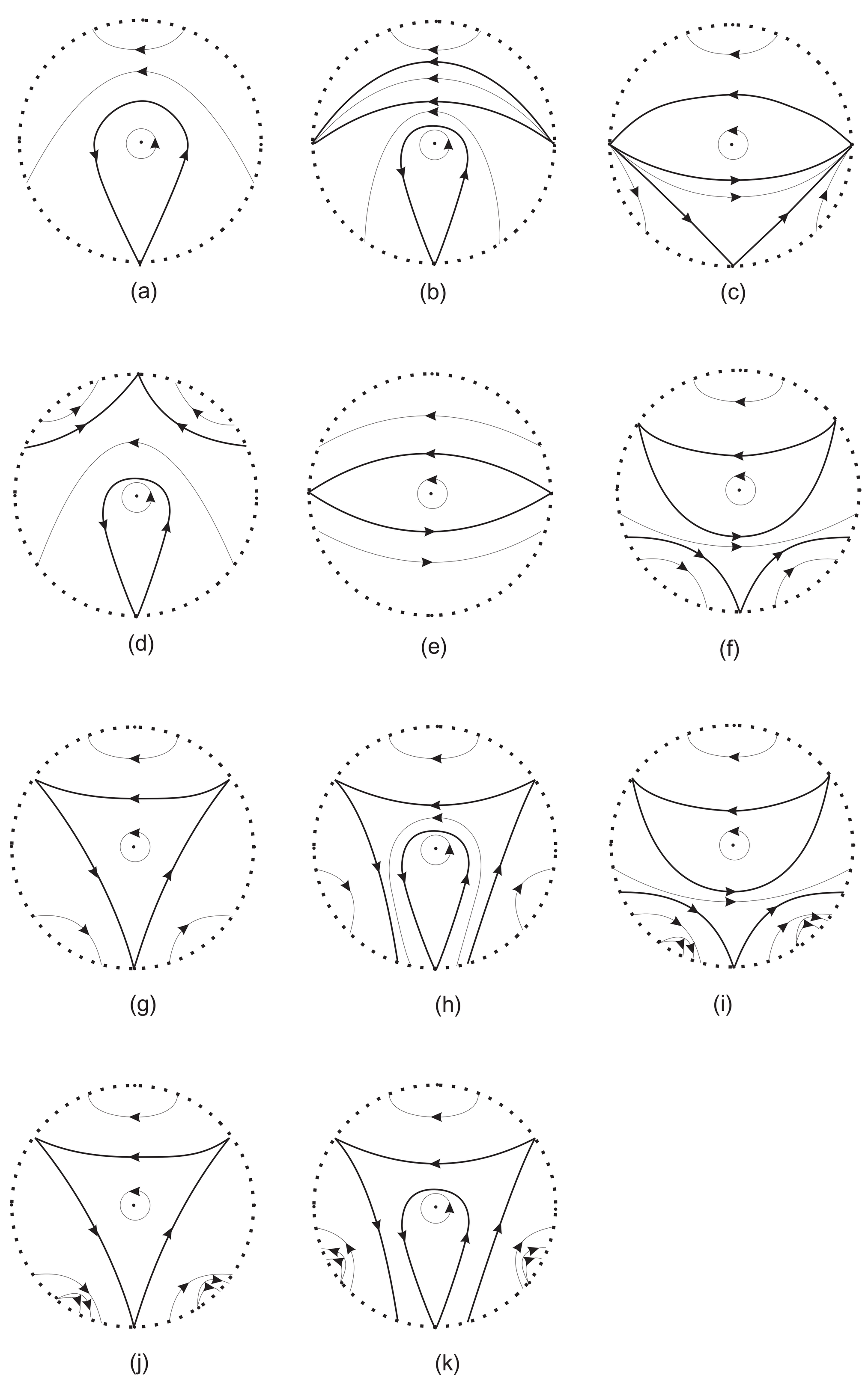
$$\begin{aligned} \dot{x} &= -y + xf(x,y), \\ \dot{y} &= x + yf(x,y) \end{aligned} \qquad (1)$$

where $f(x,y)$ is a polynomial of degree 3 with $f(0,0) = 0$. We classify the global phase portraits in the Poincaré disc of the vector fields associated to system (1).

We split our study into two cases, according to the form of $f(x,y)$ of system (1).

Our result for the quartic uniform isochronous centers (1) such that their nonlinear part is not homogeneous is the following.

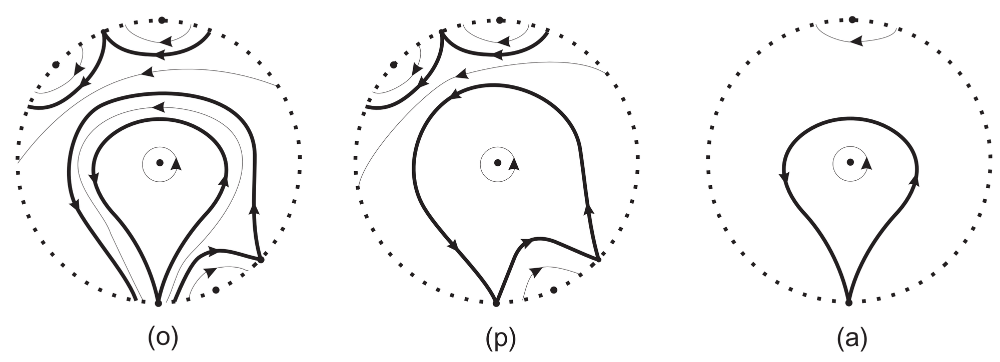
**Theorem 1.** *[2] Let (1) be a differential system with a uniform isochronous center at the origin and such that its nonlinear part is not homogeneous. Then the global phase portrait of (1) is topologically equivalent to one of the following 11 phase portraits:*



(a) (b) (c) (d) (e) (f) (g) (h) (i) (j) (k)

In the case of the quartic uniform isochronous centers (1) with homogeneous nonlinear part our result is

**Theorem 2.** *[3] Let $f(x,y)$ be a cubic homogeneous polynomial in system (1). Then (1) always has a uniform isochronous center at the origin and its global phase portrait is topologically equivalent to one of the following 3 phase portraits:*



(o) (p) (a)

Note that the phase portrait (a) in Theorems 1 and 2 are topologically equivalent.

The proofs of Theorems 1 and 2 are based on results of normal forms and singularities theory and on the use of standard blow up techniques.

## Discussion and future works

The classification of the global phase portraits in the Poincaré disc of the quartic uniform isochronous centers consists of 13 topologically different phase portraits. In [1] we provided the global phase portraits of the cubic uniform isochronous centers.

Our next step will be to provide a similar classification for the quintic uniform isochronous centers and also investigate the bifurcation of limit cycles from the periodic solutions of these centers.

## References

[1] J. Llibre and J. Itikawa, *Limit cycles for continuous and discontinuous perturbations of uniform isochronous cubic centers*, J. Comp. and Appl. Math. **277** (2014), 171–191.

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[3] J. Itikawa and J. Llibre, *Global phase portraits of uniform isochronous centers with quartic homogeneous polynomial nonlinearities*, Preprint, 2015.

[4] W. S. Loud, *Behavior of the period of solutions of certain plane autonomous systems near centers*, Contributions to Differential Equations **3** (1964), 21–36.

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