

Moments, Compositions, and Second Melnikov function in study of closed trajectories of polynomial Abel equations

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We consider the Abel differential equation

$$y' = p(x)y^2 + q(x)y^3 \quad (1)$$

with $p(x), q(x)$ polynomials. A solution $y(x)$ of (1) is called “closed” on $[a, b]$ if $y(a) = y(b)$. Study of closed solutions of (1) is directly related to the classical Hilbert 16-th (= Smale 13-th) and the Poincare Center-Focus problems.

In the last 20 years three important algebraic-analytic structures have been connected to the Abel equations: Composition Algebra, vanishing conditions for Generalized Moments, and (very recently) vanishing conditions for the second order Iterated Integrals (the second Melnikov function).

With these new tools, a serious progress has been achieved in study of Center conditions for polynomial Abel equation (1). Basically, we get more and more indications that composition condition, in the polynomial case, provides a very close approximation to the center one. In particular, very recently F. Pakovich has shown that “parametric center” for real polynomial Abel equations is equivalent to the composition condition.

I plan to present some of these new results and some open questions, stressing the role of the second Melnikov function and its connections to the moments vanishing.