Finite cyclicity of some center graphics through a nilpotent point inside quadratic systems

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In this paper we introduce new methods to prove the finite cyclicity of some graphics through a triple nilpotent point of saddle or elliptic type surrounding a center. After applying a blow-up of the family, yielding a singular 3-dimensional foliation, this amounts to proving the finite cyclicity of a family of limit periodic sets of the foliation. The boundary limit periodic sets of these families were the most challenging, but the new methods are quite general for treating such graphics. We apply these techniques to prove the finite cyclicity of the graphic (I_{14}^1), which is part of the program started in 1994 by Dumortier, Roussarie and Rousseau to show that there exists a uniform upper bound for the number of limit cycles of a planar quadratic vector field. We also prove the finite cyclicity of the boundary limit periodic sets in all graphics but one through a triple nilpotent point at infinity of saddle, elliptic or degenerate type (with a line of zeros) and surrounding a center, namely the graphics (I_{6b}^1), (H_{13}^3), and (DI_{2b}).