Parametric center problem for the Abel equation

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Let

$$y' = p(t)y^2 + q(t)y^3 \tag{1}$$

be the Abel differential equation, where t is real and $p(t)$ and $q(t)$ are polynomials or trigonometric polynomials. Equation (1) is said to have a center on a segment $[a, b]$ if all its solutions, with the initial value $y(a)$ small enough, satisfy the condition $y(b) = y(a)$. The problem of description of conditions implying a center for (1) is closely related to the classical Poincaré center-focus problem. Namely, it was shown by Cherkas in 1976 that for homogeneous polynomials of the same degree $F(x, y)$, $G(x, y)$ the center problem for the planar system

$$\begin{cases} \dot{x} &= -y + F(x, y), \\ \dot{y} &= x + G(x, y), \end{cases}$$

reduces to the center problem for (1) for some trigonometric polynomials $p = p(\cos t, \sin t)$ and $q = q(\cos t, \sin t)$. The center problem for the Abel equation with polynomial coefficients does not correspond directly to any form of the Poincaré problem, but can be interpreted as a simplified version of this problem.

The "parametric center problem" for equation (1) is to find conditions implying that the equation

$$y' = p(t)y^2 + \varepsilon q(t)y^3,$$

has a center for any $\varepsilon \in \mathbb{R}$. Posed about fifteen years ago in the series of papers of Briskin, Francoise, and Yomdin, this problem turned out to be very constructive and resulted in a whole area of new ideas and methods, related to different kind of "moment problems". In the talk we will present the main stages of the solution of the parametric center problem in the polynomial case, and will discuss the state of the art and perspectives in the trigonometric case.