Smooth linearization of periodic maps in the plane

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A continuous map $f : \mathbb{R}^2 \to \mathbb{R}^2$ is called periodic if there is an integer $m \geq 2$ such that $f^m = f \circ \cdots^{(m)} \cdots \circ f =$ identity. The rotation of angle π and the symmetry are examples of periodic maps with $m = 2$. The rotations of angle $\frac{2k\pi}{m}$ with $1 \leq k < m$ are examples of periodic maps with $m > 2$. A classical result (Brouwer, Kerékjártó, Eilenberg) says that these linear transformations are the only periodic maps from a topological point of view. This means that for any periodic map f there exists a homeomorphism of the plane ψ such that $f = \psi \circ L \circ \psi^{-1}$ where L is a symmetry or a rotation.

In this talk we will discuss the extension of this result to the $\mathcal{C}^k$-topology. The main issue will be the construction of a $\mathcal{C}^k$-diffeomorphism ψ when f is $\mathcal{C}^k$.

Some consequences for isochronous periodic differential equations will be discussed.