A continuous map $f: \mathbb{R}^2 \to \mathbb{R}^2$ is called periodic if there is an integer $m \geq 2$ such that $f^m = f \circ \cdots \circ f = \text{identity}$. The rotation of angle $\pi$ and the symmetry are examples of periodic maps with $m = 2$. The rotations of angle $\frac{2k\pi}{m}$ with $1 \leq k < m$ are examples of periodic maps with $m > 2$. A classical result (Brouwer, KerékJártó, Eilenberg) says that these linear transformations are the only periodic maps from a topological point of view. This means that for any periodic map $f$ there exists a homeomorphism of the plane $\psi$ such that $f = \psi \circ L \circ \psi^{-1}$ where $L$ is a symmetry or a rotation.

In this talk we will discuss the extension of this result to the $C^k$-topology. The main issue will be the construction of a $C^k$-diffeomorphism $\psi$ when $f$ is $C^k$.

Some consequences for isochronous periodic differential equations will be discussed.