A\textsubscript{k} slow fast systems

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In this poster, we present studies regarding a slow fast system defined as

\[ X = \varepsilon(1 + \varepsilon f_1) \frac{\partial}{\partial x_1} + \sum_{i=2}^{k-1} \varepsilon^2 f_i \frac{\partial}{\partial x_i} - \left( z^k + \sum_{i=1}^{k-1} x_i z^{i-1} + \varepsilon f_k \right) \frac{\partial}{\partial z} + 0 \frac{\partial}{\partial \varepsilon}, \tag{1} \]

where the functions \( f_j(x_1, \ldots, x_{k-1}, z, \varepsilon) \), for \( j = 1, \ldots, k \), are smooth and vanish at the origin. The corresponding slow manifold \( S \) is given by

\[ S = \left\{ (x, z, \varepsilon) \in \mathbb{R}^{k-1} \times \mathbb{R} \times \mathbb{R} \mid \varepsilon = 0, \ z^k + \sum_{i=1}^{k-1} x_i z^{i-1} = 0 \right\}. \tag{2} \]

The manifold \( S \) is equivalently given as the critical set of an \( A_k \) catastrophe. Hence, we call (1) \( A_k \) slow fast system. The most essential information of \( X \) is contained in “the principal part”

\[ F = \varepsilon \frac{\partial}{\partial x_1} + \sum_{i=2}^{k-1} 0 \frac{\partial}{\partial x_i} - \left( z^k + \sum_{i=1}^{k-1} x_i z^{i-1} \right) \frac{\partial}{\partial z} + 0 \frac{\partial}{\partial \varepsilon}, \tag{3} \]

while the rest of the terms may be considered as a perturbation

\[ P = \sum_{i=1}^{k-1} \varepsilon^2 f_i \frac{\partial}{\partial x_i} + \varepsilon f_k \frac{\partial}{\partial z} + 0 \frac{\partial}{\partial \varepsilon}, \tag{4} \]

and thus we write \( X = F + P \).

The geometric desingularization method, or blow up (as introduced in [1]), is a successful technique that can be used to study the dynamics of (1) near the origin. However, the unknown perturbation \( P \) presents many challenges in such an analysis. To overcome some of the difficulties posed by the presence of \( P \), we prove that there exists a formal transformation \( \hat{\Phi} : \mathbb{R}^{k+1} \rightarrow \mathbb{R}^{k+1} \) that brings the \( A_k \) slow fast system (1) into its principal part \( F \) [2]. That is
\[ \dot{\Phi}_* \dot{X} = F. \] (5)

We exemplify the advantages of the proposed normal form in the case of an \( A_3 \) (cusp) slow fast system [3]. We also point out the key ingredients to understand the local dynamics of all \( A_k \) slow fast systems.

