A_k slow fast systems

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In this poster, we present studies regarding a slow fast system defined as

$$X = \varepsilon(1+\varepsilon f_1)\frac{\partial}{\partial x_1} + \sum_{i=2}^{k-1} \varepsilon^2 f_i \frac{\partial}{\partial x_i} - \left(z^k + \sum_{i=1}^{k-1} x_i z^{i-1} + \varepsilon f_k \right)\frac{\partial}{\partial z} + 0\frac{\partial}{\partial \varepsilon}, \qquad (1)$$

where the functions $f_j(x_1, \ldots, x_{k-1}, z, \varepsilon)$, for $j = 1, \ldots, k$, are smooth and vanish at the origin. The corresponding slow manifold S is given by

$$S = \left\{ (x, z, \varepsilon) \in \mathbb{R}^{k-1} \times \mathbb{R} \times \mathbb{R} \,|\, \varepsilon = 0,\ z^k + \sum_{i=1}^{k-1} x_i z^{i-1} = 0 \right\}. \qquad (2)$$

The manifold S is equivalently given as the critical set of an A_k catastrophe. Hence, we call (1) A_k slow fast system. The most essential information of X is contained in "the principal part"

$$F = \varepsilon\frac{\partial}{\partial x_1} + \sum_{i=2}^{k-1} 0 \frac{\partial}{\partial x_i} - \left(z^k + \sum_{i=1}^{k-1} x_i z^{i-1} \right)\frac{\partial}{\partial z} + 0\frac{\partial}{\partial \varepsilon}, \qquad (3)$$

while the rest of the terms may be considered as a perturbation

$$P = \sum_{i=1}^{k-1} \varepsilon^2 f_i \frac{\partial}{\partial x_i} + \varepsilon f_k \frac{\partial}{\partial z} + 0\frac{\partial}{\partial \varepsilon}, \qquad (4)$$

and thus we write $X = F + P$.

The geometric desingularization method, or blow up (as introduced in [1]), is a successful technique that can be used to study the dynamics of (1) near the origin. However, the unknown perturbation P presents many challenges in such an analysis. To overcome some of the difficulties posed by the presence of P, we prove that there exists a formal transformation $\hat{\Phi} : \mathbb{R}^{k+1} \to \mathbb{R}^{k+1}$ that brings the A_k slow fast system (1) into its principal part F [2]. That is

$$\hat{\Phi}_* \hat{X} = F. \tag{5}$$

We exemplify the advantages of the proposed normal form in the case of an A_3 (cusp) slow fast system [3]. We also point out the key ingredients to understand the local dynamics of all A_k slow fast systems.

[1] F. Dumortier and R. Roussarie. *Canard Cycles and Center Manifolds.* American Mathematical Society, **121** 1996.

[2] H. Jardón-Kojakhmetov. *Formal normal forms of A_k slow fast systems.* Submitted.

[3] H. Jardón-Kojakhmetov, H. Broer, and R. Roussarie. *Analysis of a slow fast system near a cusp singularity.* In preparation.