A revisitation of Noether's theorem in variational mechanics

Gianluca Gorni

(in collaboration with G. Zampieri (University of Verona))

Dipartimento di Matematica e Informatica, Università di Udine, Udine, Italy

We revisit Noether's theorem on the constants of motion for Lagrangian mechanical systems in the ODE case, with some new perspectives on both the theoretical and the applied side, in particular *nonlocal constants of motion* and a different version of time change.

We also show that the usual Killing-type equations can be adapted to these extended views, and that the inverse Noether theorem holds also for nonlocal constant of motion in our sense. We give examples that reappraise the usefulness of the "on-flow" solutions, as opposed to the "strong" solutions of Killing-type equations.

Applications are given to Lane-Emden equation, dissipative systems, homogeneous potentials, superintegrable systems, Maxwell-Bloch system.

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