On the wave length of smooth periodic traveling waves of the Camassa-Holm equation

Anna Geyer

(in collaboration with J. Villadelprat (Universitat Rovira i Virgili))

Departament de Matemàtiques, Universitat Autònoma de Barcelona, Barcelona, Spain

The Camassa-Holm equation

$$u_t + 2\kappa\, u_x - u_{txx} + 3\, u\, u_x = 2\, u_x u_{xx} + u\, u_{xxx}, \tag{1}$$

arises as a two-dimensional shallow water approximation of the Euler equations over a flat bottom, where $u(x,t)$ for $x \in \mathbb{R},\ t > 0$ describes the horizontal velocity component and κ is a real parameter [1,2,5]. The present work [4] deals with traveling wave solutions of the form

$$u(x,t) = \varphi(x - c\,t), \tag{2}$$

where $c \in \mathbb{R}$ is the wave speed (a complete classification of all traveling wave solutions of the Camassa-Holm equation (1) is given in [6]). More precisely, we are concerned with the wave length λ of smooth periodic traveling wave solutions of the Camassa-Holm equation. The set of these solutions can be parametrized using the wave height a (or "peak-to-peak amplitude"). Our main result establishes monotonicity properties of the map $a \longmapsto \lambda(a)$, i.e., the wave length as a function of the wave height. We obtain the explicit bifurcation values, in terms of the parameters associated to the equation, which distinguish between the two possible qualitative behaviours of $\lambda(a)$, namely monotonicity and unimodality. The key point is to relate $\lambda(a)$ to the period function of a planar differential system with a quadratic-like first integral, and to apply a criterion (see [3]) which bounds the number of critical periods for this type of systems.

[1] R. Camassa and D. D. Holm. *An integrable shallow water equation with peaked solitons*. Phys. Rev. Lett. 71:1661–1664, 1993.

[2] A. Constantin and D. Lannes. *The hydrodynamical relevance of the Camassa-Holm and Degasperis-Procesi equations*. Arch. Ration. Mech. Anal., 192:165–186, 2009.

[3] A. Garijo and J. Villadelprat. *Algebraic and analytical tools for the study of the period function.* J. Differ. Equ., 257:2464–2484, 2014.

[4] A. Geyer and J. Villadelprat. *On the wave length of smooth periodic traveling waves of the Camassa-Holm equation.* To appear in J. Differ. Equ.

[5] R. S. Johnson. *The classical problem of water waves: a reservoir of integrable and nearly-integrable equations.* J. Nonlinear Math. Phys., 10:72–92, 2003.

[6] J. Lenells. *Traveling wave solutions of the Camassa-Holm equation.* J. Differ. Equ., 217:393–430, 2005.