Classification and counting of planar quasi-homogeneous but nonhomogeneous differential systems

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The planar differential system $\dot{x} = P(x, y),\ \dot{y} = Q(x, y)$, with $P, Q \in \mathbf{C}[x, y]$ is called *quasi–homogeneous* if there exist $s_1,\ s_2, d \in \mathbf{N}$ such that for an arbitrary $\alpha \in \mathbf{R}^+$, one has that $P(\alpha^{s_1}x, \alpha^{s_2}y) = \alpha^{s_1-1+d}P(x, y)$ and $Q(\alpha^{s_1}x, \alpha^{s_2}y) = \alpha^{s_2-1+d}Q(x, y)$. The quasi-homogeneous systems have important properties (for example, all of them are integrable) and they had been studied from many different points of view (integrability, centers, normal forms, limit cycles). But so far there was not an algorithm for constructing all the quasi-homogeneous polynomial differential systems of a given degree until the results of B. García, J. Llibre and J. S. Pérez del Río in 2013 that give such algorithm and that we recall here. Using this algorithm the same authors obtain the classification of the quasi-homogeneous planar systems of degree 2 and 3 and later other authors solve the case of degrees 4 (H. Liang, J. Huang and Y. Zhao in 2014) and 5 (Y. Tang, L. Wang and X. Zhang in 2015). In our work we study the problem of the classification in a general way and we obtain the exact number of different forms of quasihomogeneus but nonhomogeneous planar differential systems of an arbitrary degree n, proving a nice relation between this number and the Euler's indicatriz.

[1] B. García, J. Llibre, and J. S. Pérez del Río, *Planar quasi-homogeneous polynomial differential systems and their integrability.* J. Differ. Equ. **255** (2013) 3185–3204.

[2] H. Liang, J. Huang, and Y. Zhao, *Classification of global phase portraits of planar quartic quasi-homogeneous polynomial differential systems.* Nonlinear Dyn. **78** (2014) 1659–1681.

[3] Y. Tang, L. Wang, and X. Zhang, *Center of planar quintic quasi-homogeneous polynomial differential systems.* Discrete Contin. Dyn. Syst. **35** (2015) 2177–2191.