Separatrix skeleton and limit cycles in some 1-parameter families of planar vector fields

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In this poster we determine the bifurcation diagram of the separatrix skeleton for the 1-parameter family $(X_m^k)_{m\in\mathbb{R}}$ in function of m, where

$$X_m^k \leftrightarrow \dot{x} = y^3 - x^{2k+1}, \dot{y} = -x + my^{4k+1}$$

and $k \geq 1$ is an arbitrary but fixed integer. Recall that the *separatrix skeleton* is the union of singularities and separatrices. Adding limit cycles to it, one obtains the extended separatrix skeleton. By the Theorem of Markus, Neumann and Peixoto the extended separatrix skeleton completely determines the topological structure of continuous planar vector fields having only isolated singularities. We prove the absence of limit cycles for sufficiently small and sufficiently large m; in these cases we thus obtain the global phase portraits of X_m^k for any integer $k \geq 1$. By the results from [2] we also obtain the complete bifurcation diagram of global phase portraits of X_m^1 for increasing m. Finally we answer the nilpotent center/focus problem for the family $(X_m^k)_m$ and the existential part of Hilbert's 16th Problem using Roussarie compactification/localization method (see [3]).

The results that we present in the poster are published in [1].

[1] M. Caubergh, *Separatrix skeleton for some 1-parameter families of planar vector fields.* To appear in J. of Diff. Equations, 2015.

[2] A. Gasull, J. García, H. Giacomini, *Bifurcation diagram and stability for a one parameter family of planar vector fields.* J. of Math. Anal. and Appl. **413** 321-342, 2014.

[3] R. Roussarie, *Bifurcations of Planar Vector Fields and Hilbert's Sixteenth Problem.* Progress in Mathematics, Birkhäuser, 1998.