Zero–Hopf bifurcations in a hyperchaotic Lorenz system *II*

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The Lorenz system of differential equations in $\mathbb{R}^3$ arose from the work of meteorologist/mathematician Edward N. Lorenz [5], who studied forced dissipative hydrodynamical systems. This system become one of the most widely studied systems of ODEs because of its wide range of behaviors. In recent times a so-called *hyperchaotic Lorenz system* was introduced; see for instance [1,4] and the references therein. These systems can be precisely definite as autonomous differential systems in a phase space of dimension at least four, with a dissipative structure, and at least two unstable directions, such that at least one is due to a nonlinearity. The hyperchaotic systems has a dynamics hard to predict or control, for this reason such systems are as well of use in secure communications systems.

In this work we study, from a dynamical point of view, the 4*–dimensional zero–Hopf equilibria* in the hyperchaotic Lorenz system. Here, a 4–dimensional zero–Hopf equilibrium means an equilibrium point with two zeros and a pair of pure conjugate imaginary numbers as eigenvalues. More precisely we study zero–Hopf bifurcations of the following hyperchaotic Lorenz system (as given in [4]):

$$\begin{aligned} \dot{x} &= a(y-x)+w, \\ \dot{y} &= cx-y-xz, \\ \dot{z} &= -bz+xy, \\ \dot{w} &= dw-xz, \end{aligned} \tag{1}$$

for appropriate choices of the parameters a, b, c and d.

We characterize the conditions of existence of zero-Hopf equilibria in system (1) and then, using the method of averaging and convenient changes of variables and parameters, we find sufficient conditions so that at the bifurcation points periodic solutions emerge. We also describe the stability of these orbits.

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