Limit cycles in planar polynomial systems

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A particular version of the 16th Hilbert's problem is to estimate the number, $M(n)$, of limit cycles bifurcating from a singularity of center-focus type. The problem to finding lower bounds for $M(n)$ for some concrete n can be done studying the cyclicity for different weak-foci or centers. Since a weak-focus with high order is the most current way to produce high cyclicity, we present systems with few monomials with the highest known weak-focus order. Christopher in [1] proved that under some assumptions the linear parts of the Lyapunov constants with respect to the parameters give the cyclicity of an elementary center. We will show a new approach, namely parallelization, to compute the linear parts of the Lyapunov constants. More concretely, it is showed that parallelization computes these linear parts in a shorter quantity of time than other traditional mechanisms. Christopher's approach can be applied also to the weak-focus case. For even n, the studied polynomial system of degree n was the one obtained by [5] where the highest weak-focus order is n^2+n-2 for $n=4,6,\dots,18$. Moreover, we provide a system which has a weak-focus with order $(n-1)^2$ for $n \le 12$. We also show by concrete examples that, in some families, this approach is so powerful and the cyclicity can be obtained in a simple computational way. To show the power of this approach, we study the cyclicity of the holomorphic center $\dot{z} = iz + z^2 + z^3 + \cdots + z^n$ under general polynomial perturbations of degree n, for $n \le 13$. We prove that the cyclicity of the holomorphic center is n^2+n-2. This result give the highest lower bound for $M(6), M(7), \dots, M(13)$ among the existing results.

References

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