Singular Initial Value Problems for Quasi-Linear Ordinary Differential Equations

Matthias Seiß[1]

[1] *Institut für Mathematik, Universität Kassel, Germany, mseiss@mathematik.uni-kassel.de*

In this talk, we will discuss the existence and non-uniqueness of solutions of intial value problems for quasi-linear ordinary differential equations where the initial condition corresponds to a singular point of the equation.
In the literature there are many publications (e.g. [2] and [3]), even monographs [1], where the authors solve this problem with analytic methods, such as fixed-point theorems or sub- and super-solutions, for quasi-linear differential equations of orders one and two. In this talk we will use for our analysis the differential geometric approach presented by Werner M. Seiler in *A Dynamical Systems Approach to Singularities of Ordinary Differential Equations*, that is, we will consider a quasi-linear differential equation as a submanifold of a jet bundle equipped with a geometric structure, the Vessiot distribution. Singular points on the differential equation are critical points for the restricted projection to the base manifold. They can be characterised by changes in the properties of the Vessiot distribution.
In the case of a quasi-linear differential equation, the Vessiot distribution is projectable to the next lower jet bundle and we will work there. In this situation, points where properties of the projected Vessiot distribution change will be called impasse points to distinguish them from singular points on the differential equation. For us a generalised solution will be an invariant one-dimensional submanifold with respect to a generator of the projected Vessiot distribution.
In this talk, we are interested in the behaviour of generalised solutions of a quasi-linear differential equation near an impasse point. More precisely, we will discuss the existence and non-uniqueness of generalised solutions starting at or going through such a point. We will place emphasis on impasse points where the projected Vessiot distribution vanishes, since they are stationary points of the corresponding dynamical system. We will apply methods from the theory of dynamical systems to examples of quasi-linear differential equations of orders one and two to analyse there the behaviour of their generalised solutions. In addition we will treat the special case of an autonomous second order quasi-linear differential equation.

References

[1] R.R. Agarwal, D. O'Regan, *Singular Differential and Integral Equations with Applications*, Kluwer, Dordrecht, 2003.

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[3] Pavol Brunovsky, Ales Cerny, Michael Winkler, *A singular differential equation stemming from an optimal control problem in financial economics*, arXiv:1209.5027.