A Dynamical Systems Approach to Singularities of Ordinary Differential Equations

Werner M. Seiler[1]

[1] *Institut für Mathematik, Universität Kassel, Germany, seiler@mathematik.uni-kassel.de*

In this talk, we will discuss singularities of differential equations (as opposed to singularities of solutions of differential equations) using a geometric approach. A differential equation is considered as a submanifold of a jet bundle and a point on this submanifold is a singularity, if it is a critical point for the restricted projection to the base manifold. Thus one may consider this as a special case of the theory of singularities of smooth maps between manifolds [1].

An important geometric structure on a differential equation is its Vessiot distribution [2] and singularities can be characterised by changes in its properties. In the case of ordinary differential equations, the Vessiot distribution is generally one-dimensional and thus locally generated by a single vector field. At a regular singularity the distribution becomes vertical (but remains one-dimensional), whereas at an irregular singularity its dimension jumps. One can show that generically any vector field that generates the Vessiot distribution outside an irregular singularity vanishes when continued to the singularity [3].

In the talk we will show how the analysis of the local solution behaviour around an irregular singularity can thus be reduced to the analysis of a stationary point of an autonomous dynamical system. We will furthermore discuss the special case of a quasi-linear system (which is dominant in applications) [4]. Here we will first show that the Vessiot distribution becomes projectable and thus the problem can be considered at a lower order. Furthermore, we will show that this fact allows for genuine quasi-linear phenomena not present in general fully non-linear equations.

References

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