Elementary and Darboux first integrals for planar polynomial vector fields

J. Llibre[1], C. Pantazi[2], S. Walcher[3]

[1] *Universitat Autònoma de Barcelona, Spain, jllibre@mat.uab.cat*
[2] *Universitat Politecnica de Catalunya, Spain, chara.pantazi@upc.edu*
[3] *Lehrstuhl A für Mathematik, RWTH Aachen, Germany, walcher@mathA.rwth.aachen.de*

It is well known (due to Singer's results) that the existence of an elementary first integral (over the field of rational functions) for a planar polynomial vector field implies the existence of a first integral of a special form and also the existence of a particular integrating factor of Darboux type. In this talk we present rather general conditions that guarantee that the existence of an elementary first integral yields to the existence of a Darboux first integral. Moreover, we provide a complete characterization of such vector fields. Additionally, we provide some exceptional cases of vector fields which admit elementary first integral constructed by algebraic functions of degree two or three.

References

[1] J. Chavarriga, H. Giacomini, J. Giné, J. Llibre: *Darboux integrability and the inverse integrating factor.* J. Differential Eqs. **194** (2003), 116–139.

[2] M. Rosenlicht: *Liouville's theorem on functions with elementary integrals.* Pacific J. Math. **24** (1968), 153–161.

[3] M.F. Singer: *Liouvillian first integrals of differential equations.* Trans. Amer. Math. Soc. **333** pp. 673–688 (1992).