Symbolic Dynamics, Mixing and Entropy in the Three-Body Problem

A. Mylläri [1], V. Orlov[2] , A. Chernin [3], T. Mylläri[1]

[1] *St. George's University, Grenada, West Indies, amyllari@sgu.edu*
[2] *Sobolev Astronomical Institute, St. Petersburg State University, Russia*
[3] *Sternberg Astronomical Institute, Moscow State University, Russia,*

We use symbolic dynamics in the equal mass free-fall three-body problem. Different methods to construct (in the process of numerical integration of trajectories) symbolic sequences allow one to demonstrate (and illustrate on the Agekian-Anosova map) mixing, estimate entropies (Shannon, Markov and others), plot binary collision curves, etc.

We use Agekian-Anosova map (see Fig. 1) for the partitioning. It was used to specify initial conditions so that to consider all possible configugurations: two bodies are placed in the points (-0.5; 0) and (0.5; 0), then to consider all possible geometric configurations, the third body should be placed inside the region D (Fig. 1). The system is projected to the region D according to the relative distances between bodies. There are six different projections possible, thus we get sequences constructed from the alphabet $\{1; 2; 3; 4; 5; 6\}$. We also used partitions of the homology region D into 3 and 4 parts, having alphabet of 3 and 4 symbols correspondingly. A second approach is to fix some dynamical states (binary encounters, triple encounters etc.) during the evolution of the triple system. We used double and triple encounters to construct two more sequences with alphabets $\{1; 2; 3\}$.

Figure 2 shows typical distribution of entropy. To illustrate sensitivity to initial conditions, one can plot a sequence of images visualizing consequently first, second, third, etc. symbol in the sequence. Example (corresponding to 24^{th} symbol in the sequence) is given on the Figure 3.

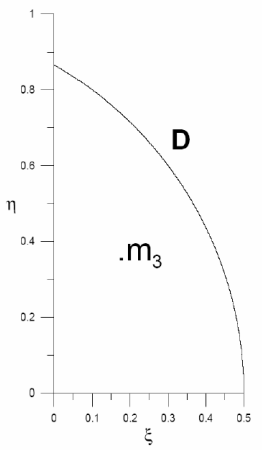

Figure 1: The Agekian-Anosova map (homology region D).

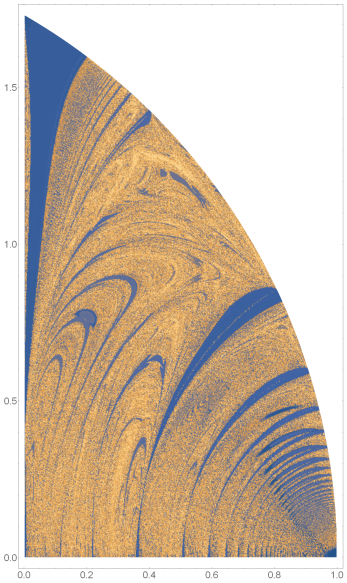

Figure 2: Values of the (Shannon) entropy in different parts of the Agekian-Anosova map are represented by different colors. Low values are shown in blue; high values are shown in light brown.

Figure 3: Sensitivity to initial conditions: different colors correspond to different symbol #24 in the symbolic sequence.