Local invariant sets of analytic vector fields

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In the theory of autonomous ordinary differential equations invariant sets play an important role. In particular, we are interested in local analytic invariant sets near stationary points. Invariant sets of the differential equation correspond to invariant ideals of the associated derivation in the power series algebra. Poincaré-Dulac normal forms are very useful in studying semi-invariants and invariant ideals. We prove that an invariant ideal with respect to a vector field, given in normal form, is already invariant with respect to the semisimple part of its Jacobian at the stationary point. This generalizes a known result about semi-invariants, that is invariant sets of codimension 1. As an application, we consider polynomial systems and bound the total degree of possible polynomial semi-invariants under some generic conditions.