

A Geometric Approach for Invariant Algebraic Curves in 2D Lotka Volterra Systems I

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In 2001, Moulin-Ollagnier [1, 2] classified all families of two dimensional Lotka-Volterra systems which have invariant algebraic curves. Twenty-five families of such curves were found using an essentially algebraic search. Our interest is to re-evaluate these families in terms of their geometric behavior. Although we are not able to provide a complete geometric proof of Moulin-Ollagnier's result, we are able to give geometric necessary conditions for the possible families of curves of low degree and, in some of these cases, can also prove the sufficiency of these conditions in purely geometric terms. The talk will be in two parts: the first will give the background and some of the tools used and the second will give more detailed examples. We will also show how this geometric understanding can be used to extend the classification of integrable critical points in Lotka Volterra systems started in [3]. This latter application is joint work with Zhaoxia Wang [4].

References

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- [4] C. Christopher, W.M.A. Hussein and Z. Wang, *On the integrability of Lotka-Volterra equations: an update*, To appear in B. Toni (ed.), *Mathematical Sciences with Multidisciplinary Applications*, Springer Proceedings in Mathematics & Statistics **157**