Greedy trajectories of Plancherel processes on two dimensional Young and Schur graphs

N. Vasilyev[1], V. Duzhin[2]

[1] *St.Petersburg department of Steklov Institute of mathematics RAS, Russia, vasiliev@pdmi.ras.ru*
[2] *St.Petersburg Electrotechnical University, Russia, vduzhin.science@gmail.com*

The sequences of the greedy branching or greedy trajectories are a special kind of infinite paths of Bratteli-Vershik diagrams provided by a certain Markov process. In these trajectories, the edge connecting two adjacent levels corresponds to the maximum transition probability of Markov process. There is an important special case of such process named the Plancherel process on 2D Young graph. The greedy trajectories of this dynamical system allows to investigate the asymptotics of the maximum dimensions of irreducible representations of symmetric group. The similar trajectories on the Schur graph allows to obtain the similar results for maximum dimensions of projective representations of symmetric group.

We present the results of a computer investigation of asymptotics for maximum dimensions of linear and projective representations of the symmetric group. This problem reduces to the investigation of standard and strict Young diagrams of maximum dimensions. We constructed some sequences for both standard and strict Young diagrams with extremely large dimensions [2]. These sequences provide the estimations of maximum normalized dimension of Young diagrams. These estimations are agreed with the Vershik conjecture [1] about the existence of the limit of maximum normalized dimensions of 2D Young diagrams. Note that this conjecture has not been proved yet. We studied the growth and oscillations of the normalized dimension function in sequences of Young diagrams. Our approach is based on analyzing the finite differences of their normalized dimensions [3]. This analysis also allows us to give much more precise estimation of the hypothetical limit constants.

There are no known exact analogue of the plancherel process for the case of 3D Young diagrams. However, there is a special process on 3D Young graph which supposedly has the property of asymptotical centrality instead of exact centrality. We present the results of computer investigation of greedy trajectories corresponding to this process. Some geometric properties of Young diagrams along these trajectories will be presented as well. Also we present a special package for manipulations with 2D and 3D Young diagrams and Young tableaux which was developed in the frame of this work.

References

[1] A. M. Vershik and S. V. Kerov. "Asymptotic behavior of the maximum and generic dimensions of irreducible representations of the symmetric group." Funktsional. Anal. i Prilozhen., 19(1):25-36, 1985.

[2] Vasilyev N. N., Duzhin V. S. Building Irreducible Representations of a Symmetric Group S(n) with Large and Maximum Dimensions. Informatsionno-upravliaiushchie sistemy [Information and Control Systems], 2015, no. 3, pp. 17-22 (In Russian). doi:10.15217/issn1684-8853.2015.1.17

[3] Vasiliev N. N., Duzhin V. S. A Study of the Growth of the Maximum and Typical Normalized Dimensions of Strict Young Diagrams, 216 Journal of Mathematical Sciences 53-64 (2016). http://doi.org/10.1007/s10958-016-2887-x