P4 and desingularization of vector fields in the plane

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Similar to the way degenerate singular points of planar curves are desingularized, so can singularities of vector fields in the plane

$$X = f(x,y)\frac{\partial}{\partial x} + g(x,y)\frac{\partial}{\partial y}$$

be desingularized. The idea is to apply a singular change of coordinates so that in the new set of coordinates, the singular point becomes more elementary and the dynamics around the singular points becomes better understandable. This idea is universal and need not be restricted to the plane, however in the plane there is a result on the finiteness of the desingularization process for analytic vector fields. This means that after a number of steps that could be carried out by a computer, the singular points is replaced by an invariant locus containing at most a finite number of semi-elementary singular points. This shows that the local study of singular points can be automated. Singular points can be found numerically in the case of polynomial X, and in that case, the study can be completed with a study at infinity by means of a so-called Poincaré compactification or Poincaré-Lyapunov compactification. We illustrate the implementation with P4 and briefly discuss P5.

References

[1] F. Dumortier, J. Llibre, J.C. Artès, *Qualitative theory of planar differential systems*, Springer-Verlag, Berlin, 2006, ISBN: 3-540-32893-9.