Search of Additional Integrals by the Normal Form Method

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Abstract

We consider a special case of the Euler-Poisson system, describing the motion of a rigid body with a fixed point. This is an autonomous sixth-order ODE system around one parameter. Among the stationary points of the system, we select two one-parameter families with fixed resonances of eigenvalues of the matrix of the linear part. At these stationary points, we compute the resonant normal form [1] of the system using a program [2] based on the MATHEMATICA package. It was shown that, if the system has an additional first integral, then its normal form is degenerated [3]. Therefore, we assume that the integrability of the ODE system can be established based on its normal form. Note however that for a full integrability of a high order system it is necessary an existence of enough numbers of first integrals.

A.D. Bruno points out conditions, which are necessary for the existence of additional first integrals [4]. These conditions are formulated as restrictions for the lowest order terms of corresponding normal forms which can be calculated analytically as functions of the system parameter.

Among the stationary points of the system, families with resonances $(0,0,i,-i,2i,-2i)$ and $(0,0,i,-i,3i,-3i)$ of eigenvalues were selected. We calculated the corresponding terms of normal forms and checked the conditions mentioned above. For mechanical values of the parameter, there are no additional first integrals except of S. Kovalevskaya and Lagrange-Poisson cases. But we found a number of values of the parameter where the normalized system has additional families of periodic solutions which are close to the stationary points. Therefore we demonstrated a use of a new approach of the integrability investigation.

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