THE PARAMETRIC RESONANCE AS A SOURCE OF CHAOTIC BEHAVIOUR IN A RESTRICTED THREE BODY PROBLEM

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In this paper, we consider the planar circular restricted three-body problem, when mass m revolves around M (M is much more than *m*) in a circular orbit and the third body is considered with negligible mass m_0 (Szebehely, 1967). We assume that all bodies move on the same plane.

The main equations for the planar circular 3-body problem in absolute cylindrical frame are:

$$\frac{d^2R(t)}{dt^2} - R(t)\left(\frac{d\lambda(t)}{dt}\right)^2 = \frac{dU}{dR}$$

$$\frac{d\left(R(t)^2\dot{\lambda}(t)\right)}{dt} = \frac{dU}{d\lambda}$$

where perturbation function in absolute frame:

$$U = \frac{Gm}{\Delta} + \frac{GM}{R}$$

where *M* – mass of the primary; m - mass of the perturbing body; *R*, *r* - distance from the mass centre test particle and perturbing body accordingly; *G* - constant of gravity, $\delta\lambda(t)=(\omega - \omega_s)t+\varphi$ - angle between perturbed and perturbing bodys (differential longitude of perturbed body), ω, ω_s - mean motions perturbed and perturbing body accordingly, φ - initial phase, and:

$$\Delta = \sqrt{R^2 + r^2 - 2Rr\cos(\delta\lambda)}$$

There are two small parameters in problem: x - radial shift from intermediate (in variations) orbit and *r/R* – the ratio of mean distance of perturbing and perturbed body. Let *R(t)=R+x(t)*, where *R* is constant. Accordingly, there are two ways of linearization (two possible sequence of expansion).

For investigation of stability, *x* may be negligible small. From instability in this case, it will be followed instability in general. So, we can restrict only 1-st order in expansion by power *x* (Mathieu's equation).

Consequently, and in this case exist the areas of instability, complying with condition:

$$\omega/\omega_s = n\Big/\left(n - 2(1 - \frac{3}{8}e^4)\right)$$

It is proved, that orbits near resonances (2n+1)/(2n-1) are instable parametric. This conclusion completely coincides with results of the studies of the declared problem in the paper Hadjidemetriou (Hadjidemetriou, 1982).

Directly, from view of linear equations in elliptic case, we can explain one interesting feature - centres of resonant zones are shifted out relative exact commensurability (resonance). In simple case one perturbing body, centres of instable resonant zones moved away from exact commensurabilities toward a source of perturbation. So, exact commensurability may be out of the relatively according instable zone!

These expressions show, that, at strong perturbations, unstable resonance zones may be overlapped. Since this, according with appearance of the chaotic motion, and when such zones fill all phase space, the behaviour of system becomes completely chaotic.

The condition of resonance overlapping is:

$$\left[(n+1)/\left(n-(1-3/4e^4)\right)-n/\left(n-(2-3/4e^4)\right)\right]=\frac{nm/M\,b_n}{2(n-2)(1-3/8e^4)}$$

Here b_n is a Laplace coefficient.

It is seems, that width of instability increased with eccentricity. On the other hand, distance between two nearest unstable areas (*Δω*) is decreased. Width of areas of instability quickly decreases with the growing of order n. However, for each m, M, the value of n, since of which the overlapping of unstable zones take place, is present. The resonance overlap criterion (Chiricov, 1979) affirms that whenever two resonances overlap, the corresponding resonant orbits become chaotic. For $e=0$, as it is evident, overlapping of unstable zones possible only at limit $n\rightarrow\infty$. It means, that effect of resonance overlapping of instable zones and related chaotic behaviour, strongly depends on the orbital eccentricity of probe particle *m*.

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